Finite Math - Fall 2018
Lecture Notes - 10/18/2018

Homework

- Section 4.2 - 56, 57, 61, 65, 67, 71

Section 4.2 - Systems of Linear Equations and Augmented Matrices

Example 1. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcl} 2x_1 & - & 3x_2 & = & 6 \\ 3x_1 & + & 4x_2 & = & \frac{1}{2} \end{array}$$

Solution. *Begin by writing the augmented matrix, then just write the equivalences at every step*

$$\left[\begin{array}{cc:c} 2 & -3 & 6 \\ 3 & 4 & \frac{1}{2} \end{array}\right] \overset{\frac{1}{2}R_1 \to R_1}{\sim} \left[\begin{array}{cc:c} 1 & -\frac{3}{2} & 3 \\ 3 & 4 & \frac{1}{2} \end{array}\right]$$

$$\overset{-3R_1+R_2 \to R_2}{\sim} \left[\begin{array}{cc:c} 1 & -\frac{3}{2} & 3 \\ 0 & \frac{17}{2} & -\frac{17}{2} \end{array}\right]$$

$$\overset{\frac{2}{17}R_2 \to R_2}{\sim} \left[\begin{array}{cc:c} 1 & -\frac{3}{2} & 3 \\ 0 & 1 & -1 \end{array}\right]$$

$$\overset{\frac{3}{2}R_2+R_1 \to R_1}{\sim} \left[\begin{array}{cc:c} 1 & 0 & \frac{3}{2} \\ 0 & 1 & -1 \end{array}\right]$$

So $x_1 = \frac{3}{2}$ *and* $x_2 = -1$.

Example 2. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcl} 5x & - & 2y & = & 11 \\ 2x & + & 3y & = & \frac{5}{2} \end{array}$$

Solution. $x = 2, y = -\frac{1}{2}$

Example 3. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcl} 2x & - & y & = & 4 \\ -6x & + & 2y & = & -12 \end{array}$$

Solution. *Begin by writing the augmented matrix, then just write the equivalences at every step*

$$\left[\begin{array}{cc|c} 2 & -1 & 4 \\ -6 & 3 & -12 \end{array}\right] \overset{\frac{1}{2}R_1 \to R_1}{\sim} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 2 \\ -6 & 3 & -12 \end{array}\right]$$

$$\overset{6R_1+R_2 \to R_2}{\sim} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 2 \\ 0 & 0 & 0 \end{array}\right]$$

The bottom row contains all zeros. This means that the system is dependent and there are infinitely many solutions. Looking at the remaining equation

$$x - \frac{1}{2}y = 2$$

we can solve for x and get

$$x = \frac{1}{2}y + 2.$$

Setting $y = t$ for a parameter t, we get $x = \frac{1}{2}t + 2$ and so the solutions to this problem are points $\left(\frac{1}{2}t + 2, t\right)$ for any real number t.

Example 4. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcr} 2x_1 & - & x_2 & = & -7 \\ x_1 & + & 2x_2 & = & 4 \end{array}$$

Solution. $x_1 = -2, x_2 = 3$

Example 5. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcr} -2x_1 & + & 6x_2 & = & 6 \\ 3x_1 & - & 9x_2 & = & -9 \end{array}$$

Solution. *For a parameter t, a solution is $x_1 = 3t - 3, x_2 = t$.*

Example 6. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcr} 2x_1 & - & x_2 & = & 6 \\ 4x_1 & - & 2x_2 & = & -1 \end{array}$$

Solution. *No solution*

Example 7. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcr} 2x & + & y & = & 1 \\ 4x & - & y & = & -7 \end{array}$$

Solution. $x = -1, y = 3$

Remark 1. *We mentioned above that the final form an augmented matrix with* exactly one solution *should look like*

$$\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array}\right]$$

If the system has infinitely many solutions, it takes the form

$$\left[\begin{array}{cc|c} 1 & m & n \\ 0 & 0 & 0 \end{array}\right]$$

and if it has no solution, it takes the form

$$\left[\begin{array}{cc|c} 1 & m & n \\ 0 & 0 & p \end{array}\right]$$

where $p \neq 0$.

Example 8. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcr} x & - & 4y & = & -2 \\ -2x & + & y & = & -3 \end{array}$$

Solution. $x = 2, y = 1$

Example 9. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcr} 2x & - & 3y & = & -2 \\ -4x & + & 6y & = & 7 \end{array}$$

Solution. *No solution*

Example 10. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcr} 0.3x & - & 0.6y & = & 0.18 \\ 0.5x & - & 0.2y & = & 0.54 \end{array}$$

Solution. $x = 1.2, y = 0.3$

Example 11. *Solve the system using an augmented matrix*

$$\begin{array}{rcrcr} 2x & - & 4y & = & 2 \\ -3x & + & 6y & = & -3 \end{array}$$

Solution. *For a parameter t, the solution is $x = 2t + 1, y = t$*